Designing a Neural-Network-based Sliding Mode Control with Extended Kalman Filter for Nonlinear Chaotic Systems

Babak Ranjbar, Behrooz Rezaie

***Abstract*—**In this paper, a neural-network-based sliding mode controller is used for stabilizing in a nonlinear chaotic system in presence of uncertainty and disturbance. Existence of noise in the systems and also inaccessibility of the system states will lead the system in a way that the neural-network-based sliding mode controller cannot operate precise enough and the existence of noise will result in unwanted effects on the output of the system. As a result, an extended Kalman filter is used for estimating the states of the system to let the proposed controller be able to control the process on a wide range of variation. This proposed method is robust against uncertainty and disturbance and shows fast and precise performance even in presence of noise. Simulation result using MATLAB software shows that the designed controller with extended Kalman filter is able to stabilize the nonlinear chaotic system in presence of undesired factors.

***Keywords*—**Sliding mode control, neural-network-based control, extended Kalman filter, Uncertainty and disturbance, Noise.

I. Introduction

Sliding mode (SM) controller is a well-known control method for the systems with uncertainty in their models or external disturbance implied to the system [1]-[5]. In recent years, SM control method has been combined with intelligent methods such as neural network (NN) [6]-[9] and fuzzy logic system (FLS) [10]-[15]. One of great advantages of such methods are their ability in controlling the complex systems, even without knowing the precise mathematical model. However, the states of the system are not usually accessible or there exists unavoidable noise in the measured states. These problems may affect the performance of the SM controllers. Therefore, utilizing observers can resolves such problems.

Observers in definitive systems or filters in random systems have wide range of usages in order to control the systems by using output feedback method [16]. The problem of nonlinear system’s state estimation is of great importance in many applications, because the states are not available in many systems and they should be estimated. A classical method for estimation problem in definitive systems is the Luenberger observer. Although this observer is very simple and effective, it has many problems in facing with random noise or with indefinite inputs, so they do not operate properly estimate the states. Furthermore, Kalman filter (KF) has solved the estimation problem in random linear systems as a linear filter by use of minimum mean square error [17]. While using KF, it is supposed that the system parameters, covariance of process and calculation noises and also system inputs are all known [18]. On the other hand, if the system is nonlinear with white noises, extended Kalman filter (EKF) can be used [19]-[20].

In this paper, a NN-based SM controller is proposed to control the system with uncertainty and disturbance. In the proposed controller, NN is used in parallel with SM controller to assist it in controlling the system and eventually decrease the error. In order to cope with the measurement noise and also for the system with inaccessible states, we also use EKF to estimate the system states. Simulation results for a nonlinear chaotic system show the effectiveness of the proposed method for controlling the system in presence of uncertainty, disturbance and noise.

The organization of the paper is as follows. In Section II, the system equations are described. Then, we present all parts of the proposed control scheme in Section III. Simulation results are given in Section IV. Finally, conclusion remarks are drawn in the last section.

II. System Description

The following equation shows a general state-space representation of a chaotic system in presence of disturbance and uncertainty:

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) & i < n \\ \dot{x}_n(t) = f(x(t)) + \Delta f(x(t)) + d(t) + u(t) \end{cases} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T \in R^n$ is the state vector, $u(t) \in R$ is the control input, $f(.)$ is a scalar nonlinear continuous function which is assumed as a known part of the model of chaotic system. Also, $\Delta f(.)$ and $d(t)$ are

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uncertainty and external disturbance respectively. In general, $\Delta f(.)$ and $d(t)$ are assumed to be bounded, i.e.:

$$\begin{aligned} &|\Delta f(.)| < \alpha \\ &|d(t)| < \beta \end{aligned} \tag{2}$$

where α and β are positive constants.

III. Proposed Method

A. Designing NN-based SM Controller

SM control is one of the most efficient robust methods for controlling the systems with uncertainty. SM control method provides some advantages such as easy and simple realization, quick response, and low sensitivity to uncertainties in the model of the system and extended disturbances.

The error is defined as:

$$e(t) = x(t) - x_d(t) \tag{3}$$

where $e(t) = (e_1(t), e_2(t), ..., e_n(t))^T \in R^n$ is the error vector and $x_d(t) = (x_{d1}(t), x_{d2}(t), ..., x_{dn}(t))^T \in R^n$ is the desired state vector. Therefore, the error dynamics can be written as follows:

$$\begin{cases} \dot{e}_i(t) = e_{i+1}(t) & i < n \\ \dot{e}_n(t) = f(x(t)) + \Delta f(x(t)) - x_d^{(n)}(t) + d(t) + u(t) \end{cases} \tag{4}$$

For tracking problem, the aim of the control design is to minimize steady state errors:

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|x(t) - x_d(t)\| \to 0 \tag{5}$$

For designing SM controller, the sliding surface is defined as the flowing equation:

$$s(t) = c^T e(t) = e_n(t) + \sum_{i=1}^{n-1} c_i e_i(t) \tag{6}$$

where $e(t) = (e_1(t), e_2(t), ..., e_n(t))^T \in R^n$ is a constant vector. With appropriate initial conditons, the tracking problem is equivalent to holding the states on sliding surface $s(t)$ for all times $t > 0$, i.e.:

$$s(t) = 0 \Rightarrow e_n(t) + \sum_{i=1}^{n-1} c_i e_i(t) = 0 \tag{7}$$

Derivating the sliding surface, we have:

$$\begin{aligned} \dot{s}(t) &= c^T \dot{e}(t) = \dot{e}_n(t) + \sum_{i=1}^{n-1} c_i \dot{e}_i(t) \\ &= f(x(t)) + \Delta f(x(t)) - x_d^{(n)}(t) + d(t) \\ &\quad + u(t) + \sum_{i=1}^{n-1} c_i e_{i+1}(t) \end{aligned} \tag{8}$$

Now, consider the following control input:

$$u_{eq}(t) = -f(x(t)) - \sum_{i=1}^{n-1} c_i e_{i+1}(t) + x_d^{(n)}(t) \tag{9}$$

Due to the presence of uncertainty and disturbance terms in the system dynamics, we need to the following control input to compensate the effects of such terms.

$$u_w = -k_w \, \mathrm{sat}(s) \tag{10}$$

where k_w is a positive control gain and sat(.) is defined as:

$$\mathrm{sat}(s) = \begin{cases} \dfrac{s}{f} & |s| < f \\ \mathrm{sign}(s) & \text{elsewhere} \end{cases} \tag{11}$$

By adding this control term, the total control input becomes:

$$u = u_{eq} + u_w \tag{12}$$

The control gain k_w must be chosen such that sliding conditions exist.

Now, to improve the performance of the control scheme, we add a control input obtained using a multi-layer NN trained by back propagation algorithm [21]. The design aim is to change the NN weights by means of descent gradient algorithm in order to minimize error [22]. Network's structure is shown in Fig. 1.

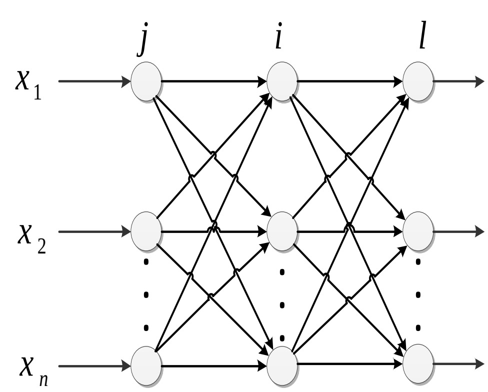

Fig. 1 The structure of multi-layer NN

As show in this figure, NN has three layers. $O_j(k) = e_j(k)$ is the output of the first layer j at sample time k and w_{ij} is the proper weights which connect the input layer j to the hidden layer i. The output of the hidden layer can be obtained as:

$$O_i(k) = \varphi\left(\sum_{j=0}^{M} w_{ij}(k) O_j(k)\right) \tag{13}$$

where M is the number of neurons in the hidden layer and $\varphi(.)$ is a excitation function which is a nonlinear continuous and differentiable function and is defined as:

$$\varphi(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{14}$$

Also, the output of the output layer l is defined as:

$$O_l(k) = \psi\left(\sum_{i=0}^{Q} w_{li}(k) O_i(k)\right) \tag{15}$$

where Q is the number of neurons in the hidden layer $\psi(.)$ is the excitation function of the output layer which is also a nonlinear continuous and differentiable function and is defined as:

$$\psi(x) = \frac{1}{2}(1 + \tanh(x)) = \frac{e^x}{e^x + e^{-x}} \tag{16}$$

It is noted here that the cost function to be minimized is defined as:

$$E(k) = \frac{1}{2} e^2(k) \tag{17}$$

B. Designing EKF

KF is extensively used for estimating state variables in noisy environment, which is based on linear state space model. The flaw related to KF is that the system must be linear. But, many physical systems are nonlinear. So, EKF is introduced to overcome this problem.

The basis of EKF is to linearize the model before implementing KF algorithm.

In this method, nonlinear system will be linearized around KF estimated point. Generally, first order Tylor series is used for linearization.

Consider the following nonlinear discretized system:

$$\begin{aligned} x_{k+1} &= f(x_k) + w_k \\ y_k &= h(x_k) + v_k \end{aligned} \tag{18}$$

The function $f(.)$ indicates a time-variant matrix function and $h(.)$ indicates au time-variant measurement matrix function. In addition, w_k and v_k are the process noises and measurement noises at time step k, respectively.

In the EKF algorithm, the state-space model of the system is linearized in every time step. After linearizing the system, EKF is implemented for estimating system states. Therefore, the estimated state obtained from EKF is an estimation around the new state. The equations of EKF is as follows:

$$\begin{aligned} \hat{x}_k &= \hat{x}_{k-1} + K_k(y_k - h(\hat{x}_{k-1})) \\ K_k &= P_{k-1} H_k^T (H_k P_{k-1} H_k^T + R_k)^{-1} \\ P_k &= (I - K_k H_k) P_{k-1} \end{aligned} \tag{19}$$

where $\hat{x}_k$ is the estimated state, and H_k, R_k, K_k and P_k are the parameters of EKF.

The controller's general block diagram with estimator is shown in Fig. 2.

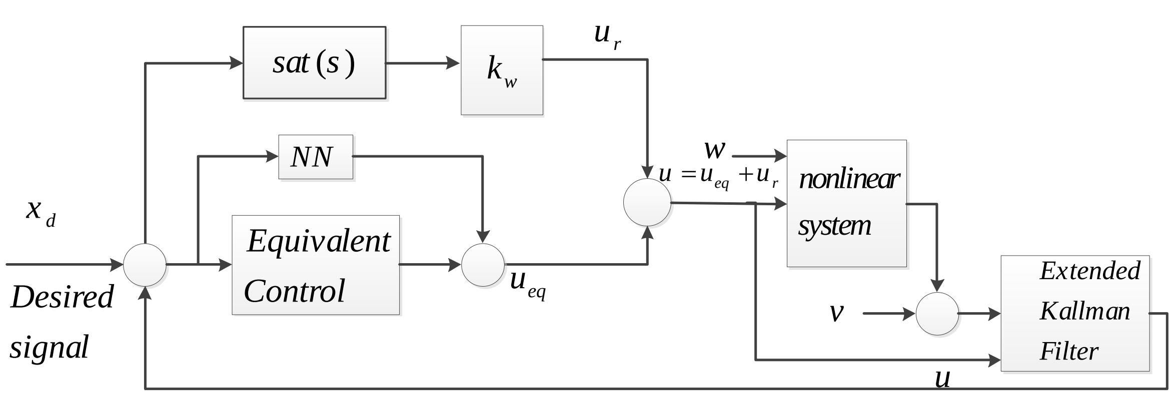

Fig. 2 Block diagram of the proposed controller and observer

IV. PUBLICATION PRINCIPLES

The following equation shows the state space equations of a chaotic system [23]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -cx_1 - bx_2 - ax_3 + x_1^2 + \Delta b(x) + d(t) + u(t) \end{cases} \tag{20}$$

where x_1, x_2 and x_3 are the system's state variables; a, b and c are the system's positive constants satisfying $ab < c$. Moreover, $X_0 = [3 \quad -4 \quad 2]^T$ is considered as the initial conditions of the system. Also, $\Delta b(x)$ and $d(t)$ are the uncertainty and disturbance, respectively and are assumed as follows:

$$\begin{aligned} &\Delta b(x) = 0.1\sin(4\pi x_1)\sin(2\pi x_2)\sin(\pi x_3) \\ &d(t) = 0.6\cos(0.86t) \end{aligned} \tag{21}$$

where $|\Delta b(x)| < \alpha = 0.1$ and $|d(t)| < \beta = 0.6$.

Fig. 3 shows phase portraits of $x_1 - x_3$ and $x_1 - x_2$ for $a = 1.2$, $b = 2.92$ and $c = 6$.

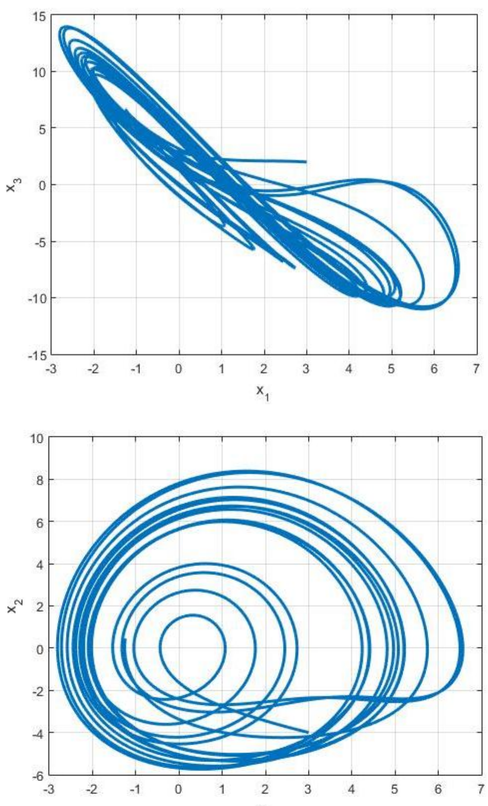

Fig. 3. Phase portraits of the system without controller

For designing the controller, sliding surface is defined as:

$$s(t) = e_3(t) + c_2 e_2(t) + c_1 e_1(t) \tag{22}$$

where $c_1 = 10$ and $c_2 = 6$. If the derivative of sliding surface considered to be equal be zero, the control input is calculated as:

$$\begin{aligned} u(t) = cx_1 + bx_2 + ax_3 - x_1^2 + x_d^{(3)} - c_1 e_2 \\ - c_2 e_3 - k_w(\text{sat}(s)) \end{aligned} \tag{23}$$

where the control gain is assumed as $k_w = 15$.

Power of the process noise and measurement noise are defined as:

$$\begin{aligned} &Cov[w_k] = 0.0001 \\ &Cov[v_k] = 0.001 \end{aligned} \tag{24}$$

Figs. 4-6 respectively indicate the system states, the control input and the sliding surface obtained by applying the proposed controller in the absence of the state estimator in presence of noise, uncertainty and disturbance. From Fig. 4, it can be seen that the noise affects the behavior of system and the proposed controller cannot operate properly.

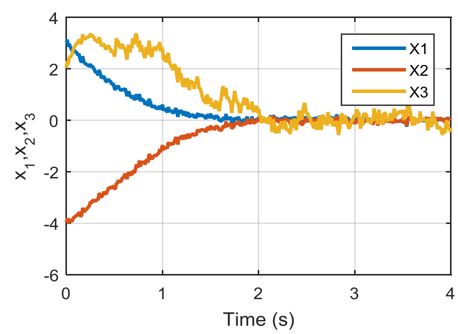

Fig. 4. System states in the presence of controller and the absence of estimator

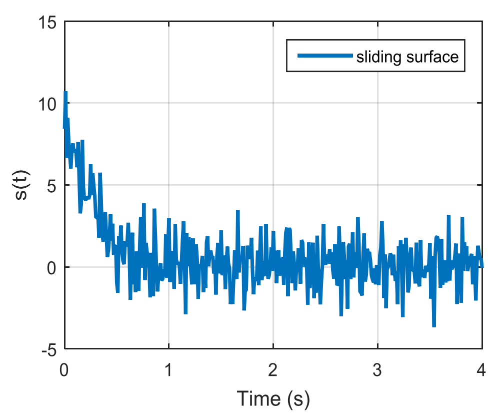

Fig. 5. Sliding surface in the presence of controller and the absence of estimator

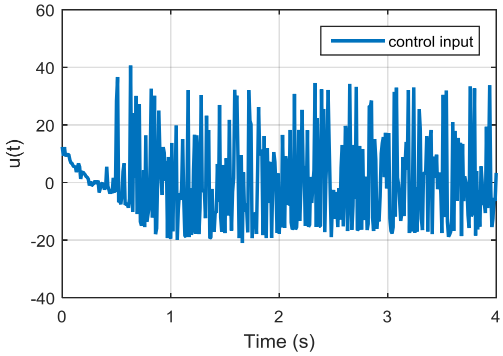

Fig. 6. Control input in the presence of controller and the absence of estimator

Now, we consider the EKF with the following initial parameters and initial estimation:

$$
\begin{aligned}
P_0 &= diag\,(2,2,2) \\
H_k &= 1 \\
x_{ekf} &= [0,0,0]^T
\end{aligned}
\tag{25}
$$

Figs. 7-9 indicate the system states, control input and sliding surface in the presence of controller and system's proposed estimated with noise and disturbance.

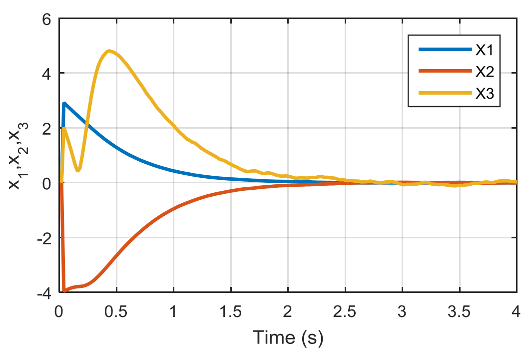

Fig.7. System states in the presence of controller and estimator

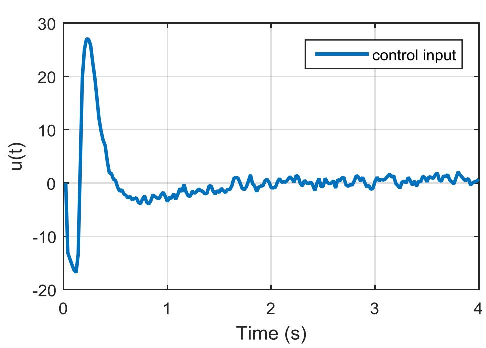

Fig. 8. Control input in the presence of controller and estimator

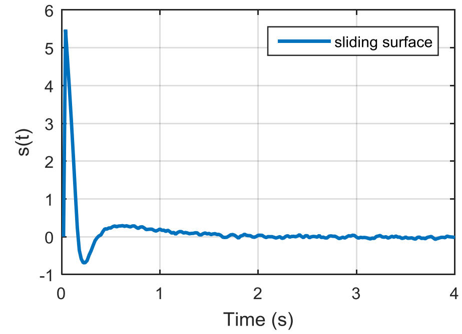

Fig. 9. Sliding surface in the presence of controller and estimator

Compared with Fig. 4, it can be observed from Fig. 7 that the proposed controller and estimators not only stabilize the system, but also minimize the effects unwanted noise.

V. CONCLUSION

In this paper, in order to control the states of a nonlinear chaotic system in the presence of uncertainty and disturbance, a NN-based SM controller has been designed. Due to existence of noise resulting in inappropriate performance of the controller, an EKF has been designed to estimate the system states. Simulation Result showed that the proposed method can successfully stabilize the chaotic system. Moreover, it decreases the estimation errors and undesired noise effects and also it is robust against uncertainty and disturbance.

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