A Modified Preventive Maintenance Model with Degradation Rate Reduction under Warranty Consideration

Chun-Yuan Cheng, Jr-Tzung Chen, Te-Hsiu Sun

Abstract—There exist different preventive maintenance (PM) models over a finite time span for deteriorating and repairable equipment. Among these PM models, it is found that a modified PM model has the lowest expected total maintenance cost and the best optimal PM policy for different types of restoration. The daily PM actions performed by production line operators, e.g. cleaning and lubrication, can reduce machine's degradation rate to a certain level (i.e., degradation rate reduction). Moreover, when considering maintenance or repair, it is often accompanied by the issue of warranty policy. The purpose of this research is to explore the optimal PM-warranty policy for the modified degradation-rate-reduction PM model with different warranty consideration. Three cases of PM-warranty policies are studied and compared in the users' point of view. In this paper, the algorithm for searching the optimal solution for each case is presented. Weibull failure Examples are given to compare the optimal solution of the three cases.

Keywords—Preventive Maintenance Model; Degradation Rate Reduction; Warranty; Finite Time Period.

I. Introduction

The aging process of a deteriorating and repairable system can be slowed down by performing preventive maintenance (PM) (Pham and Wang [1], Nakagawa [2]). Many PM models shown in literature assume the PM can restore the system to a younger age or a smaller hazard rate, such as Nakagawa [2] and Chan and Shaw [3]. However, the PM tasks, such as cleaning, adjustment, alignment, and lubrication work, may not always reduce system's age or hazard rate. Instead, this type of PM tasks may only reduce the degradation rate of the system to a certain level and is called "daily PM" in this paper. It can be seen from the literature of the reliability-centered maintenance (RCM) and the total productive maintenance (TPM) (Bertling, Allan and Eriksson [4], Zhou, Xi, and Lee [5], McKone, Schroeder and Cua [6], and Talib, Bon and Karim [7]) that the daily PM tasks is important for keeping a system or equipment in the state of high reliability. Canfield [8] proposed an infinite-time-span model for the daily PM tasks with degradation rate reduction which assumes that the daily PM can only relieve stress temporarily and slow system's degradation rate while the hazard rate is still monotonically increased. Based on Canfield's model, Park, Jung and Yum [9] and Cheng and Chen [10] developed the optimal periodic PM policy for the deteriorating systems over an infinite time span.

In real world, a system's useful life is normally finite. When an aged system is replaced by a new one, the new system seldom has exactly the same conditions (such as characteristics, investment cost, and maintenance expenses) as those of the system of the previous replacement cycle. However, not many PM models consider the conditon of finite time span. Only some examples are found, such as Pongpech and Murthy [11], Yeh and Chen [12], Cheng and Liu [13] and Ponchet, Fouladirad and Grall [14]. Hence, it is worthwhile to study the PM problem with a finite time span.

For a PM model, it can be found that a shorter time interval of PM (T) can result in a better expected total maintenance cost (TC) [15]. In literature, the "original" optimal policy of a PM model in a finite time period (L) is obtained by searching the value of T for any given number of PM (N) over the specified range $\left[L/(N+1), L/N\right)$ which has the minimal TC. However, the "original" model does limit the possibility of finding a smaller (better) TC than its optimal solution since the value of T is limited in the range of $\left[L/(N+1), L/N\right)$. Cheng et al. [15] presented a "modified" PM model with failure rate reduction by releasing the constraint of the searching range of T. It is found that the optimal PM policy of the "modified" PM model is better than the "original" PM model, such as the modified failure-rate-reduction PM model (Cheng et al. [15]), the modified age-reduction PM model [16] and the modified PM model with degradation rate reduction [17].

In real situation, equipment vendors usually provide warranty service for a new machine or system. It is worthwhile to investigate the modified PM model with different warranty consideration for the daily PM task. Therefore, the purpose of this research is to develop the optimal PM-warranty policy for the modified PM model with degradation rate reduction in a finite time span by minimizing the expected total maintenance cost. In this paper, three cases of PM-warranty models are studied in equipment owner's point of view. The first case is no warranty provided during the whole PM cycles; the second case is no PM in the warranty period and having the PM after warranty expired; the third case is having PM in the warranty period and the period after the warranty expired.

The decision variables of the modified PM-warranty model with degradation rate reduction include the PM interval (T), the

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number of PM (N) and the restoration factor (η). In this paper, we first develop the proposed three PM-warranty models. Next, we present the algorithm for searching the optimal solution which can include both fully-periodic and partially-periodic PM policies. Then, the optimal solutions of the examples with Weibull failure distribution are illustrated and compared for the three PM-warranty models. Finally, the sensitivity of the parameters to the decision variables and the expected total maintenance cost are analyzed.

II. The Modified PM-Warranty Models

The modified PM-warranty model with degradation rate reduction for the first case (i.e., no warranty provided during the whole PM cycles) is presented as a base model and the modified PM-warranty model for the second and the third cases (i.e., having warranty) are then developed and compared with the base model accordingly.

A. Nomenclature

- L The useful life time (finite time period) for the system or equipment.
- T The time interval of each PM.
- N The number of PM performed in the finite life time period (L).
- $\lambda(t)$ Failure rate function before the first PM.
- $\lambda_i(t)$ Failure rate function at time t where t is in the i^{th} PM cycle and $\lambda_0(t)=\lambda(t)$
- $\Lambda(t)$ The expected number of failures in time t without warranty or after warranty period.
- π The time interval between the N^{th} PM and L, i.e., $\pi = L - NT$.
- w The length of warranty period, where $w < L$.
- n_w The number of PM performed within the warranty period.
- η The restoration factor for measuring the level of restoration after each PM where $0 \le \eta \le 1$.
- $\tau_N(T,\eta)$ The corresponding age reduced for the degradation rate restoration in each PM when given N, which is determined by the restoration factor η and T.
- C_{mr} The minimal repair cost of each random failure.
- $C_{pm}(i,T,\eta)$ The PM cost of the i^{th} PM, which is function of i and $\tau_N(T,\eta)$.
- $TC(N,T,\eta)$ The expected total maintenance cost over the finite life time interval L, the notation is simplified as $TC(N,T)$ when $\eta = 1$.

B. The Assumptions

The following are the assumptions for the modified PM-warranty model with degradation rate reduction.

- The equipment is deteriorating over time with power law increasing failure rate (IFR) in which Weibull failure distribution is assumed in this paper, i.e.,

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \tag{1}$$

  where θ is the scale parameter and β is the shape parameter with $\beta \ge 2$.
- The system is disposed at specified finite time (L) without replacing a new one where the disposed system is assumed to have no salvage market value.
- The PM interval (T) is limited in the range of (0, L].
- The PM can reduce the equipment's degradation rate to a younger level. When given N and T, the corresponding age reduction ($\tau_N(T,\eta)$) for the degradation rate restoration in each PM is constant and is measured by

$$\tau_N(T,\eta) = \eta T. \tag{2}$$

- The minimal repair cost (C_{mr}) of each random failure is assumed to be constant.
- The cost of each PM ($C_{pm}(i,T,\eta)$) is assumed to be variable and is defined in the following equation, which is affected by equipment age (expressed by the number of PM performed) and the amount of degradation rate reduced after the PM.

$$C_{pm}(i,T,\eta) = \sum_{i=1}^{N} a + bi + c\tau_N(T,\eta) = \sum_{i=1}^{N} a + bi + c\eta T \tag{3}$$

  where a, b, and c are the coefficients of the PM cost function. For each PM, a, b, and c may imply the fixed cost, the incremental cost, and the variable unit cost of restoration effect, respectively.
- The times to perform PM and minimal repair are negligible.
- The minimal repair cost is charged to the vendor if a random failure is occurred within the warranty period.
- The PM cost is charged to the equipment owner (user) if the PM is performed in the warranty period.
- The expected total maintenance cost ($TC(\cdot)$) consists of the total minimal repair cost and the total PM cost.
- The minimal repair cost (C_{mr}) and the PM cost ($C_{pm}(\cdot)$) are assumed to include the following items: labor of repair and maintenance, material and parts, and loss of downtime in production in this paper.

C. The Failure Rate Function of the Modified PM Model with Degradation Rate Reduction

Based on Canfied [8], the failure rate function (also called the hazard rate function) of the i^{th} PM cycle (as illustrated in Fig. 1) is defined in (4). It can be seen that the $\pi < T'$ in Fig. 1 since the decision variable T' of the original PM model with degradation rate reduction is constrained in the range of $[L/(N+1), L/N)$ for a specified number of PM (N).

$$\lambda_i(t) = \begin{cases} \lambda(t), & 0 < t \le T \quad \text{for } i = 0, \\ \sum_{k=1}^{i} \left\{ \lambda\left(T + (k-1)(1-\eta)T\right) - \lambda\left(k(1-\eta)T\right) \right\} & \\ +\lambda(t - i\eta T), \quad iT < t \le (i+1)T, & \\ \qquad \text{for } i = 1,2,\ldots,N. & \end{cases} \tag{4}$$

Case 1: No Warranty but having PM in the Life Time

In the modified PM-warranty model with degradation rate reduction, the PM interval (T), the number of PM (N) and the restoration factor (η) are the decision variables. Case 1 is based on the modified PM model developed by Cheng et al. [17] (as illustrated in Fig. 2). It can be observed that the interval of π in Fig. 2 is greater than the PM interval T (i.e., $\pi \ge T$) and T has smaller value than T' of the original PM model (as shown in

Fig. 1). Cheng et al. [17] showed that the modified PM model with degradation rate reduction can provide better optimal solution than the original PM model. The expected number of random failures, $\Lambda_1(L)$, of the modified PM model in the finite time period (L) is shown as

$$\Lambda_1(L) = \sum_{i=0}^{N-1} \int_{iT}^{(i+1)T} \lambda_i(t)dt + \int_{NT}^{L} \lambda_N(t)dt, \tag{5}$$

where $\lambda_i(t)$ is define in (4).

Then, the expected total maintenance cost ($TC(\cdot)$) can be obtained as

$$TC_1(N,T,\eta) = C_{mr}\Lambda_1(L) + \sum_{i=1}^{N} C_{pm}(i,T,\eta), \tag{6}$$

where $\Lambda_1(L)$ is define in (5).

D. Case 2: No PM in the Warranty Period

Since the random failures occurred in the warranty period are repaired by the vendor, the minimal repair cost is counted after the warranty period (w). We assume that the first PM is performed at time T. The PM-warranty model for Case 2 is illustrated in Fig. 3. Hence, the expected number of random failures in the period of [w, L] is given as

$$\Lambda_2(w,L) = \sum_{i=0}^{N-1} \int_{w+iT}^{w+(i+1)T} \lambda_i(t)dt + \int_{w+NT}^{L} \lambda_N(t)dt. \tag{7}$$

The expected total maintenance cost function is shown as.

$$TC_2(N,T,\tau) = C_{mr}\Lambda_2(L) + \sum_{i=1}^{N} C_{pm}(i,T,\eta), \tag{8}$$

where $\Lambda_2(\mathrm{w},L)$ is define in (7).

E. Case 3: Having PM in the Warranty Period

In Case 3, it is assumed that the PM time interval is consistent in the entire finite time period (L), i.e., the PM is performed periodically every T unit of time and is not affected by the warranty period. Suppose the number of PM performed within the warranty period is n_w where $n_w = \lfloor w/T \rfloor$. It can be seen that $n_w = 0$ if $T > w$ and $n_w > 0$ if $T \le w$. The PM-warranty model for Case 3 is illustrated in Fig. 4.

The expected number of random failures which are occurred in the time period [w, L] is obtained in (9).

$$\Lambda_3(w,L) = \int_{w}^{(n_w+1)T} \lambda_{n_w}(t)dt + \sum_{i=n_w+1}^{N-1} \int_{iT}^{(i+1)T} \lambda_i(t)dt + \int_{NT}^{L} \lambda_N(t)dt. \tag{9}$$

Then, the expected total maintenance cost function is presented in (10).

$$TC_3(N,T,\tau) = C_{mr}\Lambda_3(\mathrm{w},L) + \sum_{i=1}^{N} C_{pm}(i,T,\eta), \tag{10}$$

where $\Lambda_3(\mathrm{w},L)$ is define in (9).

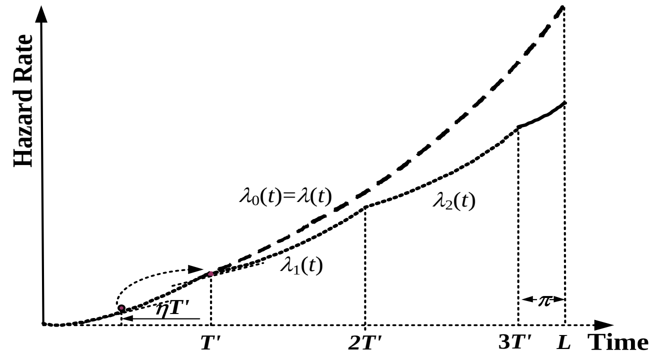

Fig. 1. The illustration of the original PM model with degradation rate reduction.

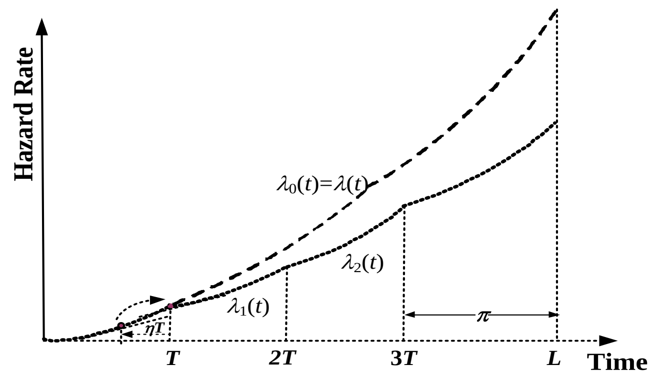

Fig. 2. The illustration of the modified PM model with degradation rate reduction (also Case 1).

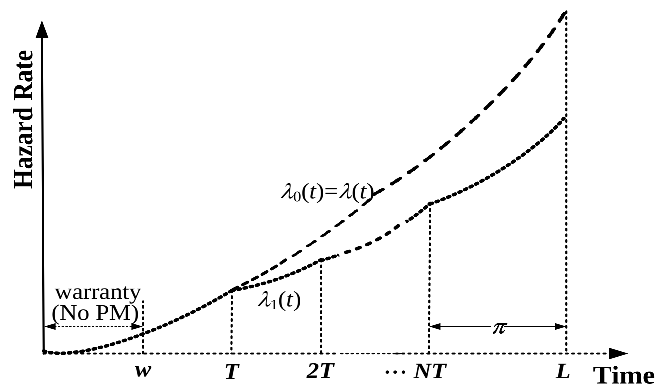

Fig. 3. The illustration of the modified PM-warranty model for Case 2.

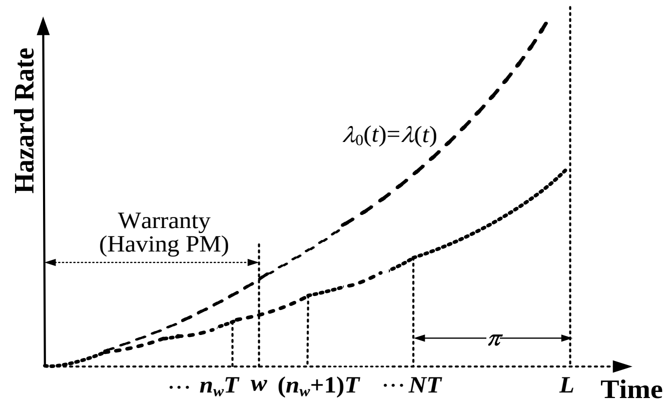

Fig. 4. The illustration of the modified PM-warranty model for Case 3.

III. The Optimal PM-Warranty Policies

The optimal solution for the above PM-warranty models can be obtained by minimizing the expected total maintenance cost $TC(\cdot)$. It requires an algorithm with numerical method to search the optimal solution. The algorithm proposed by Cheng and Liu [13] is applied in this paper since it is suitable for the modified PM models having more than one decision variable. The algorithm for Case 2 is presented below. The algorithms for Cases 1 and 3 are similar to the algorithm for Case 2 and are omitted.

A. *The Algorithm for Searching the Optimal Solutions of Case 2*

1) Let $N=0$, $T_N=L$, $\eta_N=0$.
2) Calculate $C_{\min}=TC_2(N,T_N,\eta_N)$ using (8). (Note: $TC_2(\cdot)$ is the expected total maintenance cost of no PM being performed in Case 2.)
3) Let $N=1$.
4) Calculate $T_U=(L-w)/N$.
5) Use Nelder-Mead method to Search the values of T_N in the range of (0, T_U] and η_N in the range of [0, 1] such that $TC_2(N,T_N,\eta_N)$ shown in (8) is minimized;
   let C_0 = minimal value of $TC_2(N,T_N,\eta_N)$.
6) If $C_0 \ge C_{\min}$ then
   obtain the optimal solution $N^*=N-1$, $T^*=T_{N^*}$, $\eta^*=\eta_{N^*}$, and $TC_2(N^*,T^*,\eta^*)$;
   stop,
   else
   let $N=N+1$ and $C_{\min}=C_0$;
   go to Step 4).

B. *Numerical Examples*

Numerical examples for the three cases of the modified PM-warranty PM are performed based on the following conditions: L = 5; w = 2; C_{mr} = 1; and $C_{pm}(\cdot)$ is defined in (3) where different values are assigned to the coefficients a, b, and c as shown in Tables I to III. The time between failures of the equipment is assumed to follow Weibull distribution with scale parameter θ =1 and shape parameter β = 2.5 and 3.

Table I shows the optimal solutions of the examples for Case 1 (i.e., no warranty); Table II is for Case 2 where no PM is assumed in the warranty period; Table III is for Case 3 where PM may be performed in the warranty period. It can be investigated from these tables that the optimal restoration factor is $\eta^*=1$. It means that the corresponding age reduced for the optimal degradation rate restoration is T in each PM.

It can be noted that the optimal solutions of Case 2 (no PM in warranty period) have larger total maintenance cost ($TC(\cdot)$) than Case 1 and 3 for all the examples. It is due to the property of the modified PM model, i.e., the shorter PM interval (T), the better total maintenance cost ($TC(\cdot)$). The result also means that no matter there is a warranty or not, the daily PM task (with degradation rate reduction) is necessary.

Since the PM cost shown in (3) is affected strongly by the number of PM executed and the coefficient b, the optimal values of N^*, T^*, and $TC(\cdot)$ are strongly sensitive to the coefficient b. It can be seen from the Comparing Tables II and III (which are having PM within the whole life time (L)), it can be seen that Case 1 has better $TC(\cdot)$ than Case 3 only when $b=0$. This implies that the warranty may not be necessary if the daily PM has very low cost. From the examples, it is obvious that the Case 3 is the best PM-warranty policy for the conditions of higher PM cost.

In the paper, the analysis of variance (ANOVA) is performed to show the sensitivity of parameter β, and the coefficients a, b, and c of the PM cost to the total maintenance cost ($TC(\cdot)$) which is shown in Table IV. It can be found that the coefficients a, b, and c of the PM cost do affect the total maintenance cost $TC(\cdot)$; β is sensitive to $TC(\cdot)$ for Cases 2 and 3.

TABLE I: The optimal solutions of the examples for Case 1.

Parameter			***N***		***T***		***η***		***TC***	
a	***b***	***c*** \ ***β***	2.5	3	2.5	3	2.5	3	2.5	3
0	0.5	0	4	6	0.7094	0.5636	1	1	32.8498	33.1226
0	0.5	0.5	4	6	0.6803	0.5549	1	1	34.2391	34.8003
0	0.5	0.8	4	6	0.6637	0.5499	1	1	35.0455	35.7946
0	0.8	0	3	5	0.8599	0.6467	1	1	35.5873	38.0604
0	0.8	0.5	3	5	0.8246	0.6372	1	1	36.8504	39.6652
0	0.8	0.8	3	5	0.8043	0.6317	1	1	37.5834	40.6169
0.5	0	0	15	17	0.2512	0.2381	1	1	23.7777	17.8537
0.5	0	0.5	14	17	0.2554	0.2328	1	1	25.6051	19.8548
0.5	0	0.8	14	16	0.2493	0.2426	1	1	26.6649	21.0323
0.5	0.5	0	4	5	0.7094	0.6467	1	1	34.8498	36.0604
0.5	0.5	0.5	4	5	0.6803	0.6372	1	1	36.2391	37.6652
0.5	0.5	0.8	4	5	0.6637	0.6317	1	1	37.0455	38.6169
0.5	0.8	0	3	5	0.8599	0.6467	1	1	37.0873	40.5604
0.5	0.8	0.5	3	5	0.8246	0.6372	1	1	38.3504	42.1652
0.5	0.8	0.8	3	5	0.8043	0.6317	1	1	39.0834	43.1169
0.8	0	0	10	13	0.3530	0.3004	1	1	27.3968	22.2609
0.8	0	0.5	10	13	0.3385	0.2943	1	1	29.1251	24.1935
0.8	0	0.8	10	13	0.3303	0.2909	1	1	30.1282	25.3347
0.8	0.5	0	4	5	0.7094	0.6467	1	1	36.0498	37.5604
0.8	0.5	0.5	4	5	0.6803	0.6372	1	1	37.4391	39.1652
0.8	0.5	0.8	3	5	0.8043	0.6317	1	1	38.1834	40.1169
0.8	0.8	0	3	4	0.8599	0.7597	1	1	37.9873	41.9722
0.8	0.8	0.5	3	4	0.8246	0.7494	1	1	39.2504	43.4812
0.8	0.8	0.8	3	4	0.8043	0.7433	1	1	39.9834	44.3768

IV. Conclusion

In this paper, three modified PM-warranty models with degradation rate reduction (for daily PM) are developed. We can have the following conclusion from this study: (1) the daily PM task is necessary no matter if there is a warranty provided; (2) the warranty may not be necessary if the daily PM has very low cost; (3) Case 3 (i.e., having warranty and the daily PM whin whole life time) is the best PM-warranty policy for the conditions of higher PM cost; (4) the coefficients a, b, and c of the PM cost affect the total maintenance cost.

TABLE II: THE OPTIMAL SOLUTIONS OF THE EXAMPLES FOR CASE 2.

Parameter			*N*		*T*		η		*TC*	
a	***b***	***c*** \ β	2.5	3	2.5	3	2.5	3	2.5	3
0	0.5	0	1	3	0.9808	0.5541	1	1	48.6128	101.1316
0	0.5	0.5	1	3	0.8375	0.5351	1	1	49.0665	101.9483
0	0.5	0.8	1	3	0.7601	0.5241	1	1	49.306	102.4249
0	0.8	0	1	2	0.9808	0.7101	1	1	48.9128	102.9127
0	0.8	0.5	1	2	0.8375	0.6879	1	1	49.3665	103.6117
0	0.8	0.8	1	2	0.7601	0.6751	1	1	49.606	104.0205
0.5	0	0	2	7	0.6999	0.3000	1	1	48.2559	97.8200
0.5	0	0.5	2	7	0.591	0.287	1	1	48.8996	98.8467
0.5	0	0.8	2	7	0.5342	0.2798	1	1	49.2368	99.4418
0.5	0.5	0	1	3	0.9808	0.5541	1	1	49.1128	102.6316
0.5	0.5	0.5	1	3	0.8375	0.5351	1	1	49.5665	103.4483
0.5	0.5	0.8	1	3	0.7601	0.5241	1	1	49.806	103.9249
0.5	0.8	0	1	2	0.9808	0.7101	1	1	49.4128	103.9127
0.5	0.8	0.5	1	2	0.8375	0.6879	1	1	49.8665	104.6117
0.5	0.8	0.8	1	2	0.7601	0.6751	1	1	50.106	105.0205
0.8	0	0	2	5	0.6999	0.3880	1	1	48.8559	99.6291
0.8	0	0.5	1	5	0.8375	0.3727	1	1	49.3665	100.5796
0.8	0	0.8	1	6	0.7601	0.3641	1	1	49.606	101.1322
0.8	0.5	0	1	3	0.9808	0.5541	1	1	49.4128	103.5316
0.8	0.5	0.5	1	2	0.8375	0.6879	1	1	49.8665	104.3117
0.8	0.5	0.8	1	2	0.7601	0.6751	1	1	50.106	104.7205
0.8	0.8	0	1	2	0.9808	0.7101	1	1	49.7128	104.5127
0.8	0.8	0.5	1	2	0.8375	0.6879	1	1	50.1665	105.2117
0.8	0.8	0.8	0	2	-	0.6751	1	1	50.406	105.6205

TABLE III: THE OPTIMAL SOLUTIONS OF THE EXAMPLES FOR CASE 3.

Parameter			*N*		*T*		η		*TC*		n_w	
a	***b***	***c*** \ β	2.5	3	2.5	3	2.5	3	2.5	3	2.5	3
0	0.5	0	4	5	0.7455	0.6	1	1	28.9095	30.612	2	3
0	0.5	0.5	4	5	0.7125	0.6	1	1	30.3671	32.112	2	3
0	0.5	0.8	4	5	0.6939	0.6	1	1	31.2109	33.0121	2	3
0	0.8	0	3	4	0.9101	0.75	1	1	31.2875	34.8594	2	2
0	0.8	0.5	3	4	0.8761	0.75	1	1	32.6273	36.3594	2	2
0	0.8	0.8	3	4	0.8555	0.75	1	1	33.4064	37.2594	2	2
0.5	0	0	11	12	0.2727	0.25	1	1	22.2184	20.3439	7	8
0.5	0	0.5	11	12	0.2727	0.25	1	1	23.7184	21.8438	7	8
0.5	0	0.8	11	12	0.2727	0.25	1	1	24.6184	22.7438	7	8
0.5	0.5	0	4	5	0.7455	0.6	1	1	30.9095	33.112	2	3
0.5	0.5	0.5	3	5	0.8761	0.6	1	1	32.3273	34.612	2	3
0.5	0.5	0.8	3	5	0.8555	0.6	1	1	33.1064	35.5121	2	3
0.5	0.8	0	3	4	0.9101	0.75	1	1	32.7875	36.8594	2	2
0.5	0.8	0.5	3	4	0.8761	0.75	1	1	34.1273	38.3594	2	2
0.5	0.8	0.8	3	4	0.8555	0.75	1	1	34.9064	39.2594	2	2
0.8	0	0	8	10	0.375	0.3	1	1	25.0116	23.606	5	6
0.8	0	0.5	8	10	0.375	0.3	1	1	26.5166	25.106	5	2
0.8	0	0.8	8	10	0.375	0.3	1	1	27.4116	26.006	5	2
0.8	0.5	0	3	5	0.9101	0.6	1	1	31.8875	34.612	2	6
0.8	0.5	0.5	3	5	0.8761	0.6	1	1	33.2273	36.112	2	6
0.8	0.5	0.8	3	5	0.8555	0.6	1	1	34.0064	37.0121	2	6
0.8	0.8	0	3	4	0.9101	0.75	1	1	33.6875	38.0594	2	3
0.8	0.8	0.5	3	4	0.8761	0.75	1	1	35.0273	39.5594	2	3
0.8	0.8	0.8	3	4	0.8555	0.75	1	1	35.8064	40.4594	2	3

TABLE IV: THE SENSITIVITY ANALYSIS OF THE TOTAL MAINTENANCE COST.

Source	Case 1		Case 2		Case 3	
	F value	p -value	F value	p -value	F value	p -value
β	.605	.441	36755.832	≈0.000	27.510	≈0.000
a	12.517	≈0.000	7.998	0.001	22.707	≈0.000
b	214.879	≈0.000	35.288	≈0.000	301.836	≈0.000
c	5.997	0.005	4.500	0.017	12.367	≈0.000

REFERENCES

[1] H. Pham and H. Wang, "Invited review: imperfect maintenance," *European Journal of Operational Research*, vol. 94, pp. 425-438, 1996. http://dx.doi.org/10.1016/S0377-2217(96)00099-9

[2] T. Nakagawa, "Periodic and sequential preventive maintenance policies," *Journal of Applied Probability*, vol. R-23/2, pp. 536-542, 1986. http://dx.doi.org/10.2307/3214197

[3] J.K Chan and L. Shaw, "Modeling Repairable Systems with Failure Rates that Depend on Age and Maintenance," *IEEE Transactions on Reliability*, vol. 42, pp. 566-571, 1993. http://dx.doi.org/10.1109/24.273583

[4] L. Bertling, R. Allan, and R. Eriksson, "A reliability-centered asset maintenance method for assessing the impact of maintenance in power distribution systems," *IEEE Transaction on Power Systems*, vol. 20, no. 1. pp. 75-82, 2005. http://dx.doi.org/10.1109/TPWRS.2004.840433

[5] X. Zhou, L. Xi, and J. Lee, "Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation," *Reliability Engineering and System Safety*, vol. 92, pp. 530-534, 2007. http://dx.doi.org/10.1016/j.ress.2006.01.006

[6] K.E. McKone, R.G. Schroeder, and K.O. Cua, "The impact of total productive maintenance practices on manufacturing performance," *Journal of Operations Management*, vol. 19, no. 1, pp. 39-58, 2001. http://dx.doi.org/10.1016/S0272-6963(00)00030-9

[7] A. Talib, B. Bon, and N. Karim, "Total Productive Maintenance application to reduce Defects of Product," *Journal of Applied Sciences Research*, vol. 7, no. 1, pp. 11-17, 2011.

[8] R. Canfield, "Cost Optimization of Periodic Preventive Maintenance," *IEEE Transactions on Reliability*, vol. R-35, pp. 78-81, 1986. http://dx.doi.org/10.1109/TR.1986.4335355

[9] D.H. Park, G.M. Jung, and J.K. Yum, "Cost Minimization for Periodic Maintenance Policy of a System Subject to Slow Degradation," *Reliability Engineering and System Safety*, vol. 68, pp. 105-112, 2000. http://dx.doi.org/10.1016/S0951-8320(00)00012-0

[10] C.Y. Cheng and M. Chen, "The periodic maintenance policy for a Weibull life-time system with degradation rate reduction under reliability limit," *the Asia-Pacific Journal of Operational Research*, vol. 25, no. 6. pp. 793-805, 2008.

[11] J. Pongpech and D.N.P. Murthy, "Optimal Periodic Preventive Maintenance Policy for Leased Equipment," *Reliability Engineering & System Safety*, vol. 91, pp. 772-777, 2006. http://dx.doi.org/10.1016/j.ress.2005.07.005

[12] R.H. Yeh and C.K. Chen, "Periodical preventive-maintenance contract for a leased facility with weibull life-time," *Quality & Quantity*, vol. 40, pp. 303-313, 2006. http://dx.doi.org/10.1007/s11135-005-8095-2

[13] C.Y. Cheng and H.H. Liu, "The finite-time-period preventive maintenance policies with failure rate reduction under a warranty consideration," Journal of the Chinese Institute of Industrial Engineers, vol. 27, pp. 81-89, 2010. http://dx.doi.org/10.1080/10170660903513616

[14] A. Ponchet, M. Fouladirad, and A. Grall, "Maintenance policy on a finite time span for a gradually deteriorating system with imperfect improvements," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 225, pp. 105-116, 2011. http://dx.doi.org/10.1177/1748006XJRR349

[15] C.Y. Cheng, R. Guo, M. Chen, and J.T. Chen, "A modified preventive maintenance model with failure rate reduction in a finite time span," *Proceedings of the 4th Asia-Pacific International Symposium on Advanced Reliability and Maintenance Modeling*, pp.121-128, Wellington, New Zealand, December 2-4, 2010.

[16] C.Y. Cheng, M. Wang, and J.T. Chen, "A Modified Age Reduction PM Model in a Finite Time Span," *Proceedings of the 2011 IEEE*

International Conference on Quality and Reliability, pp.145-149, Bangkok, Thailand, September 14-17, 2011. http://dx.doi.org/10.1109/icqr.2011.6031698

[17] C.Y. Cheng, J.T. Chen, T.H. Sun, and M.L. Liu, "A Modified Preventive Maintenance Model with Degradation Rate Reduction in a Finite Time Span," *Proceedings of the 11th International Conference on Informatics in Control Automation and Robotic*s, pp. 746-750, Vienna, Austria, September 1-3, 2014. http://dx.doi.org/10.5220/0005068607460750