Optimum Robust LQR Control for an Oil Cooler System

Changho Han, Dokyung Choi, and Seokkwon Jeong

***Abstract*—** The compressor variable speed (CVS) method used for control of vapor compression refrigeration system is described in detail as main target. This system is basically multi-input multi-output (MIMO) system. Many control design methods such as PID control based on a transfer function model, optimum control based on the state space model have been studied for dealing with the MIMO system. The PID control method is hard to predict the control performance in a design step. Also, this method have to get each transfer function for all controlled variables. Therefore, it is desirable that optimum control will be applied to the MIMO system. This paper presents mathematical modeling of an oil cooler system for optimum control based on a state space model. The mathematical model of the oil cooler is built by combining dynamic model of manipulators with differential equations of heat exchangers. The optimum control method on the basis of this model is designed to minimize an evaluation function that includes input energy and control error. The validity and robustness of the proposed optimum controller are reviewed by some computer simulations and real experiments.

***Keywords*—** Evaluation function, Linear Quadratic Regulator (LQR) , Oil cooler system, Optimum control, State space model, Robustness

I. Introduction

A compressor variable speed control method is generally applied to an oil cooler system for machine tools [1]. It is a MIMO system that simultaneously controls target temperature and superheat. Furthermore, it is necessary to consider not only control performance but also energy saving in controller design. PID control based on a transfer function model and optimal control based on the state space model have been studied as control methods of the MIMO system. As the PID controller can be designed after a real plant is configured, it is difficult to predict control performance at the design stage [2]. Also, the PID controller requires a transfer function model for each input and output on a control target. It is necessary to control fan motors in both an evaporator and a condenser to establish the most desirable control for the refrigerator cycle in the future. In this case, the modeling of the system and PID gains determining of the controllers are hard to deal with because the PID controller have to be designed considering the desired design specifications according to the four controlled variables. In addition, even though the controller is designed through this complicated process, it does not guarantee the optimal control performance and it is difficult to control effectively the interference between the target temperature and superheat [3]. Therefore, application of optimum control based on the state space model is desirable for non-interference control and optimal control performance of the MIMO system. In this paper, we build a plant model of the refrigeration cycle through its mathematical analysis and design the LQR controller which is the optimum controller based on it. Differential equations of heat exchangers in a refrigeration cycle are combined with the dynamic model of a compressor and an EEV (Electronic expansion valve) to derive a state equation and an output equation from physical meaning of the controlled variables. The controller consists of an optimal control system that simultaneously considers control error and input energy by extending to the servo control system based on the LQR. The validity of the designed controller was evaluated by simulations and experiments for the oil cooler system.

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II. State Space Modeling of Oil Cooler System

Fig. 1 shows a schematic diagram of an oil cooler system.

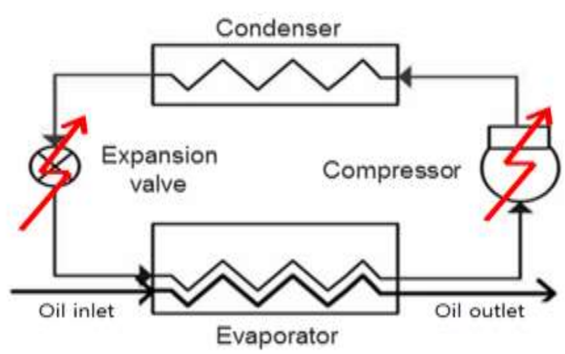

Fig. 1 Conceptual diagram of an oil cooler system.

The output variables are the oil outlet temperature T_o and superheat T_s which are controlled variables. The input variables are the compressor frequency F_i and EEV opening angle A_v, which are manipulated variables. State equations and output equations of state space models are expressed by Eq.(1) and Eq.(2) using state

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variable $x(4\times1)$, output variable $y(2\times1)$, and input variable. Where A is system matrix, B is distribution matrix of input u.

$$\dot{x} = Ax + Bu \tag{1}$$
$$y = Cx \tag{2}$$

To obtain the state equation of vapor compression refrigeration system, the Navier-Stokes Eq.(3) is applied to heat exchangers such as an evaporator and a condenser which are treated as moving boundary model (MBM) [4]. Eq.(4) is energy equilibrium equation that represents amount of heat transfer between refrigerant and secondary fluid. Eq.(5) shows the heat transfer equation which means that the amount of energy change on the tube wall of the heat exchanger is equal to the difference between the energy of the refrigerant and the external energy. In addition, the Leibniz's integral rule applies to Eq.(3), Eq.(4) and Eq.(5) to get nonlinear partial differential equations about the evaporator and the condenser.

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho u}{\partial z} = 0 \tag{3}$$
$$\frac{\partial(\rho h - P)}{\partial t} + \frac{\partial(\rho u h)}{\partial z} = A_i\alpha_i(T_w - T_r) \tag{4}$$
$$(C_p\rho V)_w\dot{T}_w = A_i\alpha_i(T_r - T_w) + A_o\alpha_o(T_a - T_w) \tag{5}$$

These nonlinear partial differential equations are linearized by applying the Taylor first order approximation around operating point, and they can be expressed as Eq.(6) by reducing dimension and removing the elements that have little impacts on the controlled variables. Moreover, the output equation of Eq.(7) is derived from physical meaning of the output variables which are oil outlet temperature T_o and superheat T_s. The system equation of the refrigeration cycle is obtained through Eq.(6) and Eq.(7).

$$\begin{bmatrix} \dot{P}_c \\ \dot{h}_c \\ \dot{P}_e \\ \dot{h}_e \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ b_{21} & 0 \\ 0 & b_{32} \\ 0 & b_{42} \end{bmatrix} \begin{bmatrix} F_i \\ A_v \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} T_o \\ T_s \end{bmatrix} = \begin{bmatrix} \dfrac{\dot{m}_r}{\dot{m}_{oil}C_{oil,p}} & -\dfrac{\dot{m}_r}{\dot{m}_{oil}C_{oil,p}} \\ -\dfrac{1}{C_p}\dfrac{h_g}{h_c} & \dfrac{1}{C_p} \end{bmatrix} \begin{bmatrix} h_c \\ h_e \end{bmatrix} \tag{7}$$

From Eq.(7), the state variables affecting controlled variables are the outlet enthalpy (h_c , h_e) of the condenser and evaporator. The dynamic characteristics of controlled variables can be determined through this system equation.

III. OPTIMUM ROBUST LQR CONTROLLER DESIGN

Fig. 2 shows block diagram of LQR servo system. It is a magnified system consisting of system equations, state feedback gain K_1, servo gain k_2, and integrator for eliminating steady state error at step input. In the LQR controller design, the solution of the optimal regulator minimizing the evaluation function expressed by Eq.(8) is obtained by the Riccati equation, and the optimum gains (K_1, k_2) are fulfilled by using it.

$$J = \int_0^\infty \left[e^T(t)Qe(t) + u^T(t)Ru(t)\right]dt \tag{8}$$

As the control error weight Q and the input energy weight R in Eq.(8) are appropriately designed, LQR control that minimizes energy consumption and control error can be realized with optimal control performance. In addition, the designed optimum servo control system follows the target value without steady-state error under the fluctuation of the parameters A, B of the system.

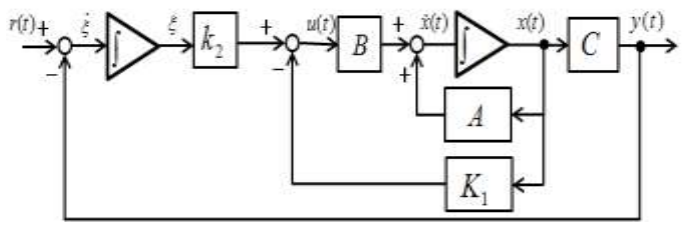

Fig. 2 Block diagram of LQR servo system.

IV. SIMULATION RESULTS AND ANALYSIS OF DYNAMIC CHARACTERISTICS

In order to verify the validity of the designed controller, the unit step input applied to the controller to figure out behaviors of controlled variables and manipulated variables. Furthermore, robustness of control system was examined through the simulation of reference variation. Fig.3 shows the indicial responses of designed controller and its manipulated variables. It was confirmed that controlled variables T_o and T_s converge to the each target value. The behaviors of the dynamic characteristics were similar to the real system.

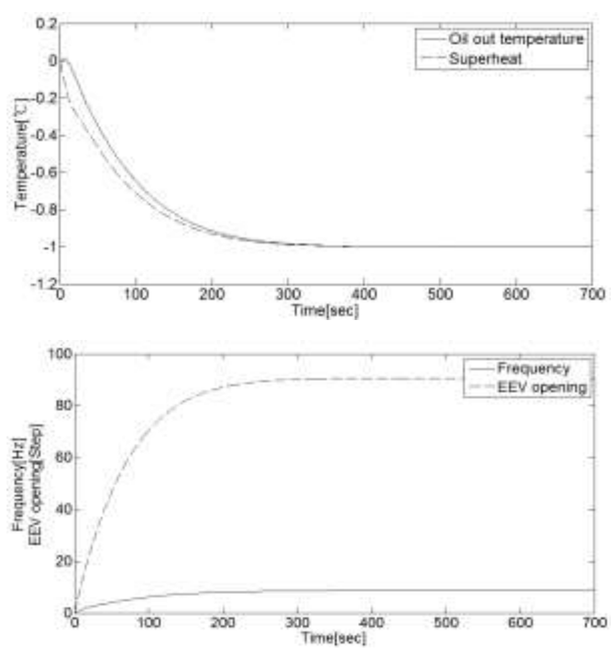

Fig. 3 Indicial responses using optimum gain. (From top to the

bottom: controlled variables, manipulated variables)

Fig. 4 shows responses of the controlled variables and behaviors of the manipulated variables when the reference is continuously changed by ±1℃. From the results of Fig. 4, it was confirmed that the controlled variables exactly converge to the each set value without steady-state error and the behaviors of the manipulated variables were also seen favorably. As a result, it is figured out the control was accurate even under the condition of the continuous reference fluctuation.

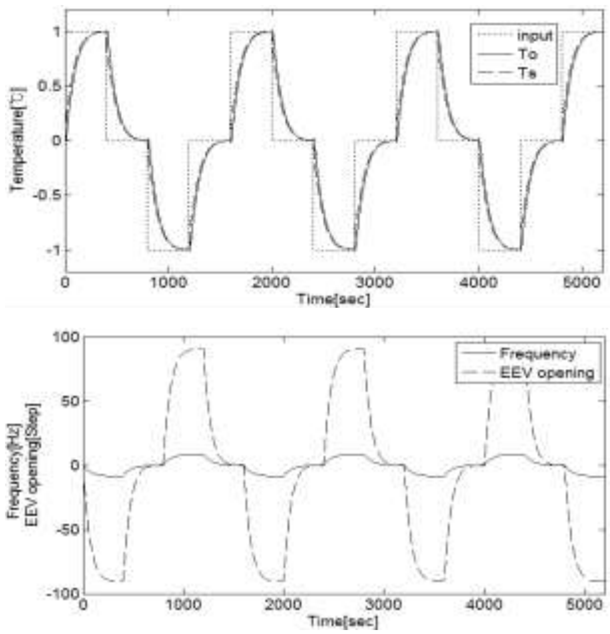

Fig. 4 Simulation results considering reference variation. (From top to the bottom: controlled variables, manipulated variables)

V. Experimental Results and Analysis

Fig. 5 indicates schematic diagram of the control system of an oil cooler. The Labview DAQ system as control device, “V/f= constant” control type inverter for compressor variable speed and step motor drive for EEV opening angle manipulator are used. The electric heater (1.5 kW) instead of a machine tool was used to impose thermal load. In addition, thermocouple and resistance thermometer PT-100 were used to acquire temperature information.

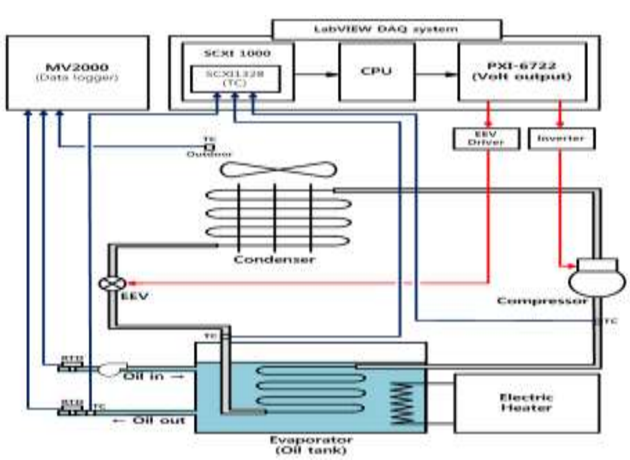

Fig. 5 Control system of an oil cooler for experiments.

Table I is specifications of the test unit, Table II shows experimental conditions respectively. The oil outlet temperature was 25℃ considering the characteristic of the machine tools. The superheat was set at 11℃ as the target value, which can maintain the optimum COP condition while preventing liquid compression phenomenon.

TABLE I: Specifications of The Test Unit

Component	Note
Compressor	Rotary type, 3 [HP]
Condenser	Air-cooled fin and tube type
Evaporator	Bare tube type
Refrigerant	R-22

TABLE II: Experimental Conditions

Item	Note
Oil flow rate	22.5 [*l/min*]
Ambient air temperature	27 ℃
Sampling time	1 [sec]
Target temperature	25 ℃

Fig. 6 shows responses of the controlled variables and the manipulated variables during the starting test. At Fig. 6.1, the oil outlet temperature was controlled at an initial temperature of 28℃, and the compressor frequency which is manipulated variables was operated at the maximum of 90 *Hz* . As the error decreased after 6o [sec], the frequency also decreased, and the temperature of the oil outlet converged to the target temperature of 25℃.

Fig. 6.2 represents superheat responses when the EEV opening angle is controlled. It was increased because of the interference effect caused by abrupt change of the frequency at start. However, it was accurately converged to the target value of 11℃ by controlling the manipulated variables.

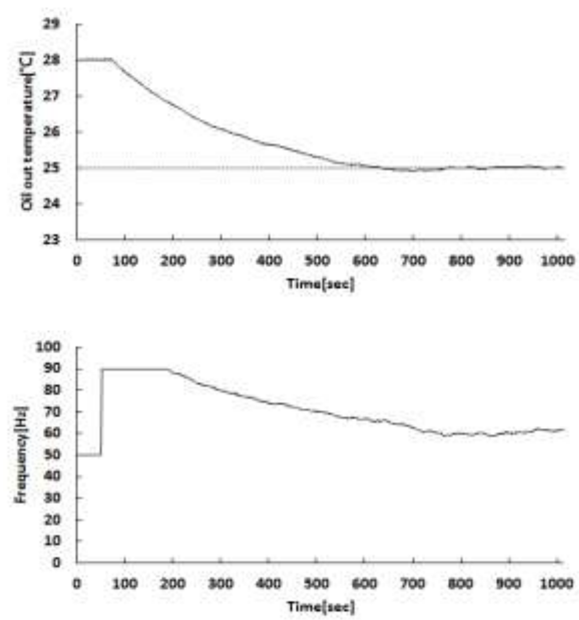

Fig. 6.1 Experimental results of starting. (From top to the bottom: oil

outlet temperature response, compressor frequency)

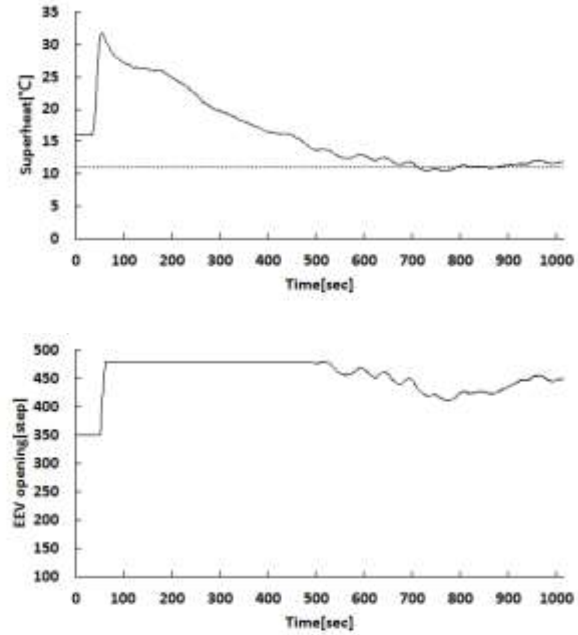

Fig. 6.2 Experimental results of starting. (From top to the bottom: superheat response, EEV opening angle)

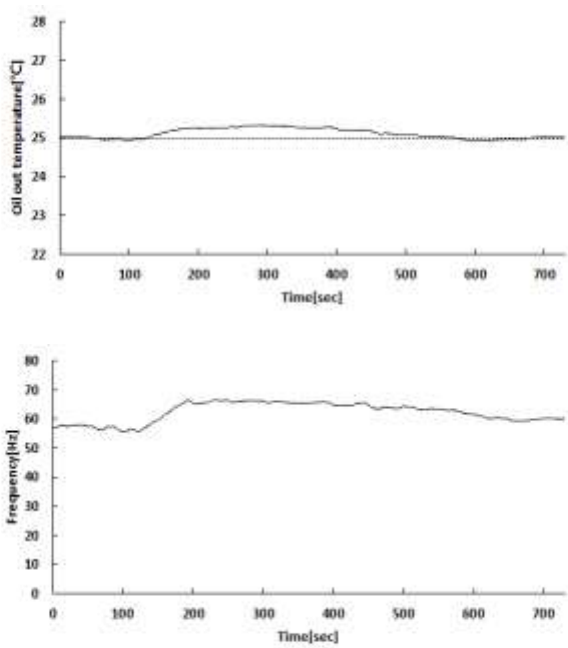

Fig. 7.1 Experimental results under thermal load variation. (From top to the bottom: oil outlet temperature response, compressor frequency)

Fig. 7 shows responses of the controlled variables when the thermal load is changed. Fig. 7.1 represents the result when the thermal load was increased by 0.1kW at 150sec. Accordingly the compressor frequency was raised from 58Hz to about 70Hz and then decreased to 60Hz, finally oil outlet temperature T_o was controlled at 25℃.

Fig. 7.2 indicates superheat responses under the thermal load change. The superheat was raised from 11℃ to around 15℃ because of the interference effects. So, to reduce the superheat, the EEV opening angle

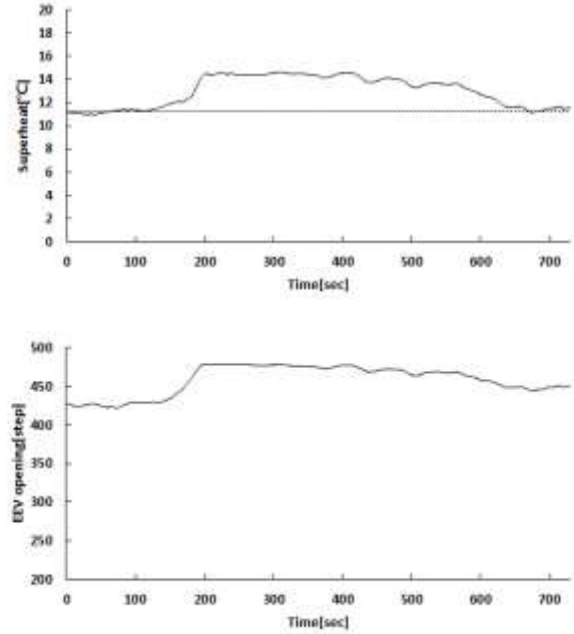

Fig. 7.2 Experimental results under thermal load variation. (From top to the bottom: superheat response, EEV opening angle)

was accelerated at 150sec. Even though the thermal load was fluctuated, the designed LQR controller was accurately operated in this condition.

VI. Conclusion

In this study, the optimum robust LQR controller based on the state space model for an oil cooler system was designed. The state equations were obtained by applying the Navier-Storkes equation to the heat exchangers of the refrigeration system such as an evaporator and a condenser treated as moving boundary model (MBM), and by combining the dynamic models of the compressor and the EEV. Additionally, the dimension of obtained state equations was lowered by removing the elements that have little impacts on the controlled variables. Therefore, the optimum robust LQR controller was designed based on the empirical state space model. In addition, the validity of the LQR controller and the proposed model were verified through the simulation and experiments. In the simulation, behaviors of the low-dimensional system was seen precisely. Furthermore, the control performance and robustness of the designed controller were demonstrated through the simulation of the indicial responses and the reference change. Also, in the experiment, the control performance of the controller was verified under the condition that the thermal load was constantly changed.

From the above results, the designed optimum controller guarantees optimal control performance. In addition, the performance of the controller can be predicted in design stage. It is also expected that this modeling method will be applied to the system based on the refrigeration cycle which requires the evaluation function such as energy saving or precision control in the future.

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